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Problems in the Design and Manufacture of Energy Storage Capacitors

by

Grenfell P. Boicourt

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PROBLEMS IN THE DESIGN AND MANUFACTURE OF ENERGY STORAGE CAPACITORS

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ABSTRACT

During the past few years much developmental work has been done to provide better energy storage capacitors for controlled thermonuclear research. One of the most serious problems in the manufacture of energy storage capacitors is a lack of understanding of the type of faults which can arise during fabrication. This report gives a detailed description of the types of faults found in various energy storage capacitors and discusses their origin, including variations in margins and active widths introduced by the winding machines. Formulas for estimating margin fields and pad overvoltages are derived.

I. INTRODUCTION

Capacitor banks are the most widely used sources of energy for fast pulsed experiments for controlled thermonuclear research (CTR). In the past ten years, personnel in the Los Alamos Scientific Laboratory CTR program have been working with the capacitor industry to develop new types of energy storage capacitors for use in these banks. During this time, much information has been obtained from the testing and disassembly of failed units.

This report results from the experience accumulated in developing 10-kV, 20-kV, 50-kV, 60-kV, and 75-kV capacitors, but most directly from the 1.85-μF, 60-kV capacitor development program for Scyllac. During this program it became obvious that one of the main problems with the units was not that the original design was faulty, but that inadequate quality control in certain areas could produce capacitor failures. This report attempts to show how this situation can be corrected. The faults discussed are real, not theoretical, and demand consideration in setting up quality control procedures. Since LASL has used capacitors from several manufacturers, it has been possible to compare units of similar design in a way not possible to the individual manufacturer. This report describes the faults found in opened energy storage capacitors and shows how the units can be improved by making use of this information. Most of the faults described come about in the manufacture of the capacitor by deviations from the ideal design; however, the material on design included here is aimed at the analysis of the effect of deviations from the ideal and cannot be considered a substitute for the basic design functions of the capacitor industry.

II. DESCRIPTION OF TYPICAL ENERGY STORAGE CAPACITOR

To discuss the faults in an energy storage capacitor, it is necessary first to describe a typical unit and to give some details of its manufacture.

The 2-μF, 50-kV, low-inductance capacitor shown in Fig. 1 is used as an example. Figure 2 is a cutaway view of the unit. The packs (F) which make up the capacity of the unit, are attached with scrub solder to a collector web (D) at the top. A

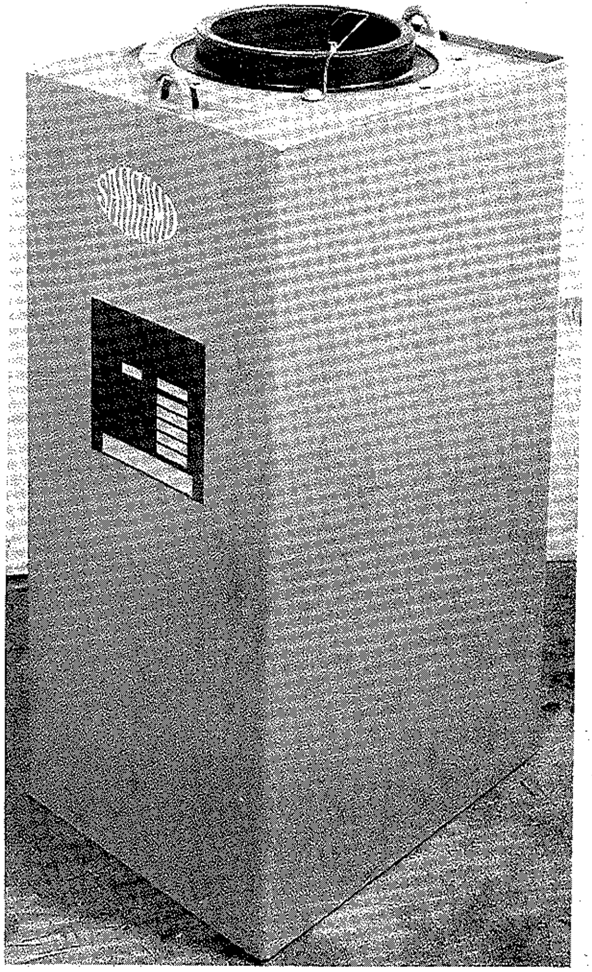

Fig. 1. 2-μF, low-inductance energy-storage capacitor.

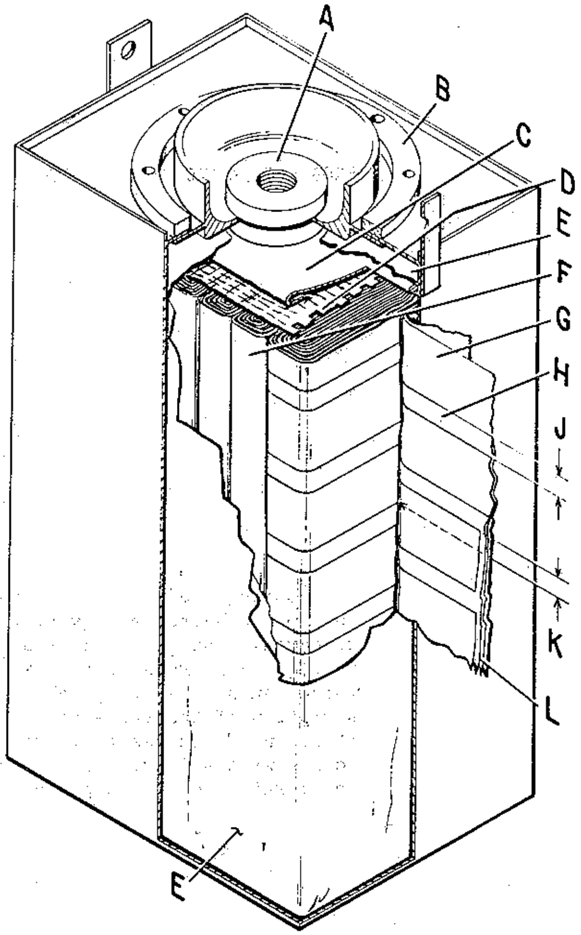

Fig. 2. Sketch of typical energy-storage capacitor.

similar connection occurs at the bottom. The collector web is soldered to the outer case at the bottom of the unit and to the collector plate (C) at the top. The collector plate is attached to the center electrode stud (A) which forms one of the output connections of the capacitor. The case forms the other output connection through the outer flange ring (B). This type of connection minimizes the inductance of the capacitor.

A multiple-section winding is used for all capacitors intended for operation above 10 kV, and this is the type of pack which will be described here. Figure 2 also shows a sketch of the pack construction. A single layer of the winding, called a sandwich, is made up of one layer of insulation, one set of foils, another layer of insulation, and the opposing set of foils. The foils (G in Fig. 2) form the conducting plates of the capacitor. Only the two end foils are connected to the external conductors. Foils are separated by several sheets of oil-impregnated Kraft paper, called the pad, that forms the active dielectric of the capacitor (Fig. 3(a)). The area by which one foil overlaps an opposite foil determines, along with the thickness of the paper, the capacity of one section of the multisection capacitor. This area is termed the active area, and the width of overlap is called the active width. The active width is shown as K in Fig. 2. The foil is separated from the adjacent foils on the same side of the paper by a margin (J in Fig. 2).

In Fig. 3(a) the W_i are the active widths, the M_i are the margins, the paper pad is shown as solid black, and the foils are shown as hatched. The electrical circuit is shown in Fig. 3(b).

The pack is made on a winding machine as follows. Rolls of paper are set up on the winding machine and several turns, consisting of a sheet from each roll, are taken on the winding mandril. The number of

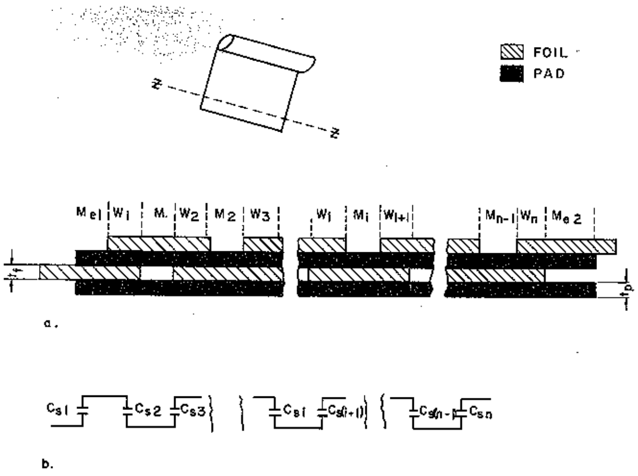

Fig. 3. (a) Schematic layout edge-on view across one layer of a capacitor pack Z - Z. (b) Equivalent circuit of pack.

rolls of paper is equal to twice the number of plies required between the foils. The foils are then introduced, one set being separated from the opposing set by the required number of sheets of paper. For example, in a pack using a pad with six layers of paper, the first set is inserted between the third and fourth sheets of paper and the second set is inserted between the ninth and tenth sheets. When the winding mandril is turned, the two outside sets of three sheets come together to form a six-sheet pad. The pack is then wound on the mandril until a predetermined diameter is reached. The foils are cut off and several more turns, consisting of paper only, are made. The wound pack is taken from the winding machine and flattened. The proper number of packs for the capacitor are assembled and the assembly is inserted in the case. The packs are then scrub-soldered to the collector web (D), following which the case is sealed except for the fill holes. The fill holes are left open for vacuum removal of moisture and for impregnation of the capacitor. The unit is then heated under vacuum to drive out any moisture in the paper and the case. After drying, the impregnant is introduced into the unit under vacuum at an elevated temperature. When impregnation is complete, the unit is cooled and the fill holes are sealed. The unit is now ready for use. Ideally, the margins and active widths will all be equal and uniform for the entire length of the winding, all the solder connections will be made properly, there will be a minimum of residual moisture, the impregnant will be of the proper purity, and the unit will have a long and useful life.

III. OBSERVED DEPARTURES FROM THE IDEAL CAPACITOR

Most faults occur in materials and manufacture.

A. Materials. Materials of interest are impregnant, dielectric, and foils.

1. Impregnant. The impregnant most commonly used for energy storage capacitors is castor oil, a vegetable product of variable properties. (Silicone oils are attractive but are too expensive for general use). Castor oil used in capacitors must have a lower acid number than is usually achieved by the supplier, and reprocessing by the capacitor manufacturer is necessary. Because oxidation products in castor oil degrade its dielectric properties, the oil must be handled under vacuum or under an inert gas. Improperly processed castor oil can be detected by its color and smell. Good oil is virtually odorless, clear, and almost colorless. Partially oxidized or overheated oil will have an amber color. Oil from a new capacitor, if it has an odor, must be considered suspect. Oil from a capacitor used for more than a few thousand discharges will always have an odor. If the oil was initially good, the odor will not be too noticeable, except in units run a very long time, and will have a heavy nonspecific character. In an inadequately ventilated room, the gaseous products from the oil may cause allergic reactions characterized by a runny nose and by sneezing. If the oil was initially bad, the odor will be sharp and pungent, somewhat like newmown weeds. These methods of detection are quite unscientific; however, they work.

2. Dielectric. The paper dielectric can deviate from optimum in several ways. The thickness of the paper can vary. As delivered to the capacitor manufacturer the allowed variation is 20%. If all sheets in the pad were at the allowed minimum, the electric field in the dielectric would be increased to 125% of the design value. The paper will also have small holes. The effects of the thickness variation and holes are minimized by using several layers of paper in the pad since it is unlikely that all sheets will be at the minimum thickness or that several holes will line up.

Another problem is included impurities in the paper, most commonly insects and felt particles. Figure 4 shows an insect found in a pad. The number of insects in paper produced by some manufacturers is high and although, to our knowledge, no one has

Fig. 4. Enlarged photograph of insect found in paper dielectric.

investigated the dielectric properties of castor oil-impregnated insects, our opinion is that the dielectric strength is lower than that of the paper. If the insect is large, the body may fall out of the paper, leaving a hole. We have observed holes 1/4 in. in diameter with blood extending for about 1/8 in. around the periphery. The felt comes from part of the paper-making apparatus over which the paper is drawn. Small bits at a low rate of incidence must be tolerated, but a high rate of incidence would be cause for rejection of the paper. A further problem is nonuniform width. Overwidth paper is not serious, but underwidth paper will reduce the effective width of the outside margins. This results in an overvolting of the margins that may lead to failure of the pack.

3. Foils. The problem of nonuniform widths occurs also with the foils. A change in foil width changes the active area, therefore the capacity of a section in the pack is changed. This may produce a severe overvoltage of one section of a pack (see Sec. IV). Tears and punctures in the foil may cause problems of two types. A tear or small puncture is usually accompanied by a tab of foil loose on three sides. If this tab is folded back on itself, it is seen by the foil opposite it as a high point and a locally increased electric field results. If the tab becomes wadded, it may puncture the dielectric and produce a shorted section.

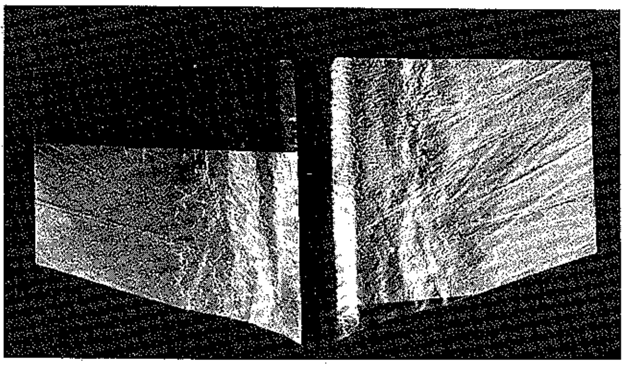

Fig. 5. Foil break just beginning to burn due to high-current arcing.

The second type of problem is one frequently overlooked. A break in a foil represents a poor electrical connection. At the high currents usually required from energy storage capacitors, current passing the break may cause sparking and burning. The resulting damage to the dielectric can lead to failure of the pack. Figure 5 illustrates this phenomenon. The foil on the left was broken close to the upper edge. Carbonization can be seen on the left side of the break. Carbonization can also be seen on the paper opposite the break. This is shown on the right, where the pad has been pulled open like a book. The spot shown had not yet produced a short.

A third type of foil problem arises from careless making and cutting of the foil. Figure 6 shows an improperly made foil where the center part of the foil was missing for several feet. Figure 7 shows the result when a piece of this same foil crossed a

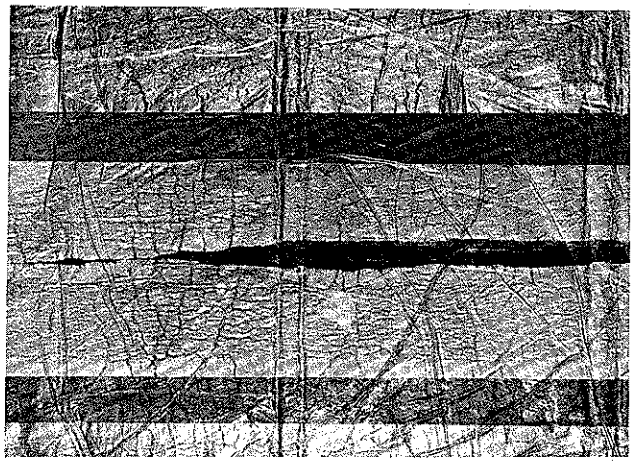

Fig. 6. Improperly made foil.

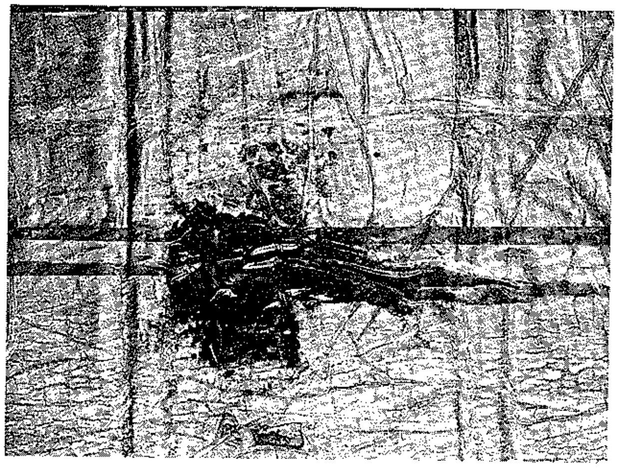

Fig. 7. Failure produced when foil in Fig. 6 crossed margin.

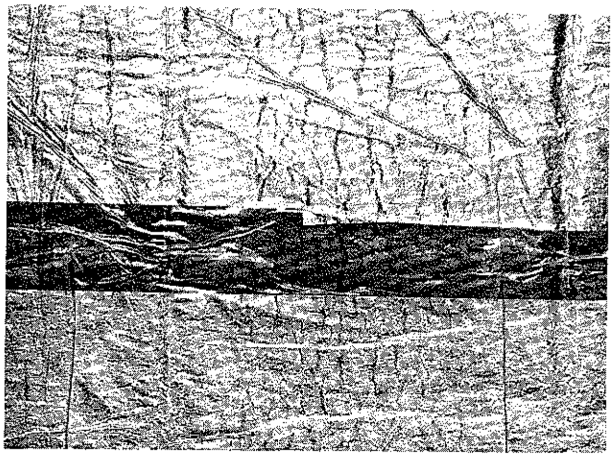

Fig. 8. Improperly cut foil with strip folded back.

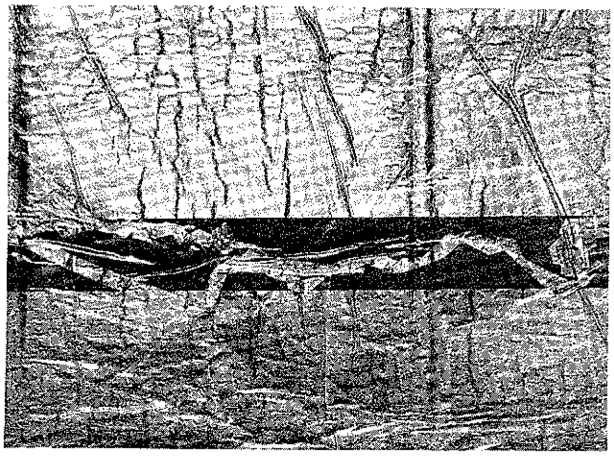

Fig. 9. Two sections shorted by foil strip in Fig. 8.

margin. Figures 8 and 9 show what can happen if the foil is improperly cut. Figure 8 shows the point where a bad cut was made and the shred of foil was allowed to remain in the roll. During winding the shred was folded back and trailed out through the winding. Figure 9 shows the result: two sections shorted out about 8 feet further down the winding.

B. Manufacture

1. Winding Machine Setup. The setup of the winding machine determines the individual margin widths and the foil active widths of each capacitor section if it is assumed that the foil does not wander during winding. A margin smaller than the pack design value may result in a failure across a margin and a margin greater than the pack design value usually means a narrow margin elsewhere in the pack. In the ideal pack, all interior margins are the same width.

The active width is determined by the amount that one foil overlaps the foil on the opposite side of the pad. Too small an active width results in a small capacity section, and since the voltage divides inversely as the capacity, the voltage appearing across this section will be higher than the design value. As an example, an active width 10% low will result in a voltage 12% high across the section, with all other sections assumed to be perfect.

Careful alignment of the paper is necessary during setup because small misalignments reduce the effective end margins and lower the paper's mechanical resistance to tearing during the soldering of the end foils.

From observation of many packs, a 10% variation from the design values is common. A detailed evaluation of the effect of these variations is given in Sec. IV.

2. Paper and Foil Wander. Paper and foil wander refers to the back and forth, sidewise drift of the individual sheets of paper or foils during winding. If the foil drifts, the result can be a locally narrow margin and a possible loss of one, several, or all plies of insulation at the edge margins. If all plies are missing, a shorted section results; if only a few plies are missing, the pad is weakened and usually the effective margin is narrowed. Both types of wander can result in a fold-over. (Fig. 10).

Figure 11 shows what happens when a foil wanders across an edge margin. The margin should have been even wider than it is shown on the left side.

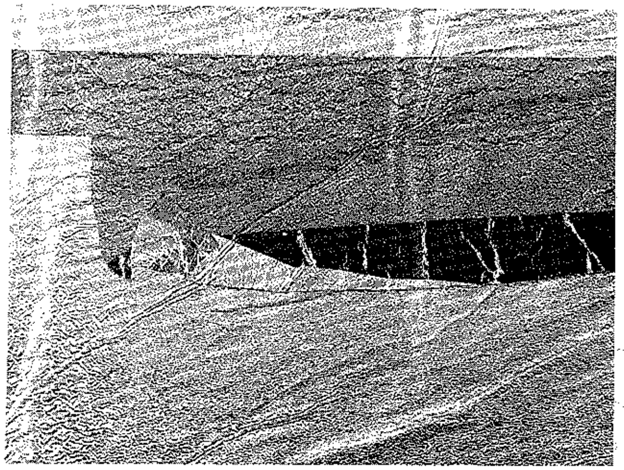

Fig. 10. Foil tear and start of a fold-over.

In this case, it appears that the entire margin was lost farther to the right but the pad is too burned to tell. The foil has also been broken and restarted in this region. This will be considered in Sec. III.

3. <u>Fold-Over of Paper and Foil</u>. Fold-over refers to the folding back on itself of a paper or a foil. This can occur if there is a local slack area in the material, if it is necessary to realign the material to correct wander, or if the winding or holding mandrils are not properly aligned. A fold-over may cause no problem if it is flat and does not weaken the pad or reduce the margin. If two folds intersect, the point of intersection may be raised enough to produce a sharp point which could injure the dielectric. If the fold-over in a foil is of any appreciable length, the active area of the section is reduced, lowering the section capacity and increasing the section voltage. Figure 10 shows the start of such a fold-over. The most fatal result of this type is a paper fold-over at the margin which narrows the edge margin. This is a surprisingly common fault as can be seen from Table I.

4. <u>Wrinkles and Folds</u>. Wrinkles refer to non-straight line distortions in a single layer of wrap. Folds refer to straight-line creases in either a single layer or through several or all layers of the pack.

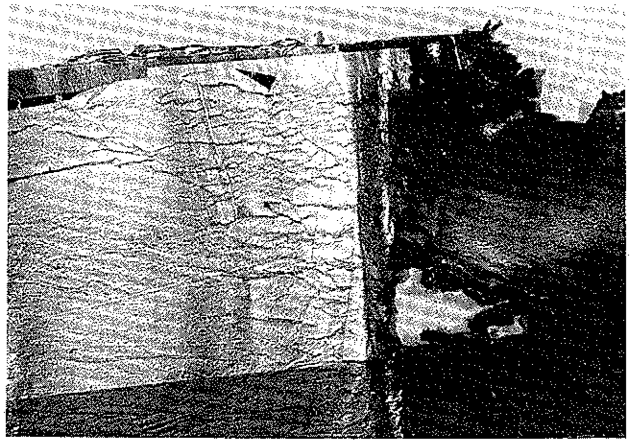

Fig. 11. Failure caused by careless restart of broken foil. Note narrowed edge margin.

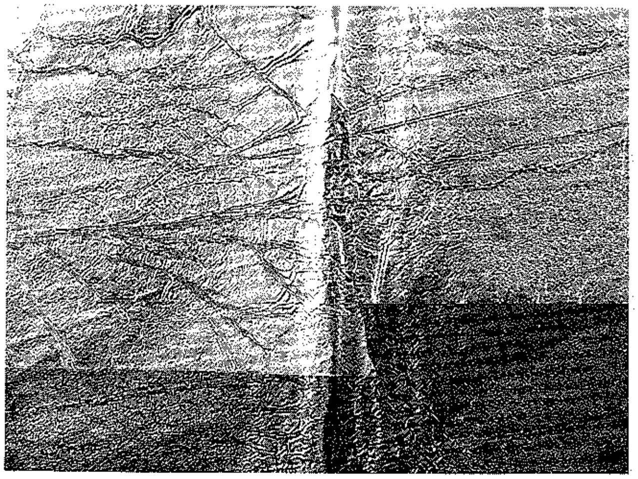

Fig. 12. Restart of broken foil. Note sharp point projecting into margin.

Wrinkles arise from two main causes. The first, and usually less severe cause, is due to shrinkage and expansion of the paper during drying and impregnation. The other cause is misalignment of the winding mandril. Wrinkles are evident in Figs. 5 through 12. Severe wrinkling can cause weak spots or tears in the dielectric pad, which leads to failure. Folds are produced from the initial compression of the round winding to a flat pack, from placing the flat pack on the edge of a table, or from the hold-down bar of the winding mandril. The folds can be classified as flat, sharp, or knife, the last being a reentrant fold.

Flat folds do not seem to contribute to capacitor failure, but sharp or knife folds probably do by local weakening of the pad. These folds would thus be connected with failures occurring after a considerable number of shots. Folds on the margin seem to act as capillary pipes for carbon if there is any carbonized oil from poor solder connections. In this case, carbon can be sucked across the edge margin and may cause a short.

5. <u>Winding Starts</u>. When beginning a new pack, the

winding machine operator first runs on a short length, usually 2 to 4 ft, of paper and then starts to feed in the foil. The foils must be started carefully. The foils must line up with the set-up rolls; otherwise, fold-overs occur during winding until the margins do line up. Misalignment here also results in narrowed margins. Second, the ends of the foils must tail out straight because if they fold over, the folded part presents a local narrowed margin and a sharp point to the foil opposite it across a margin. In severe cases, the folded part can reach across the margin to short out a section of the pack. Even if the foils do not fold, they may curve, and cases have been found where the foil tail curved over an 18-in. arc and shorted with the next foil in the set.

6. Foil and Paper Breakage. During winding of a pack, a foil or a sheet of paper may break. If this is not replaced, a defective pack will result. If a foil is partly missing, the two sections of the pack associated with this foil will have lowered capacity with a consequent increase in voltage across the sections. Even if the broken foil is restarted, aluminum chaff from the broken edge can be left in the region of the break or restart and may drift into a margin, causing high electric field points and an increased average field across the margin. When the foil is restarted, care must be taken to make the joint smooth because, if the ends of the foils are not aligned properly, a sharp point may project across the margin. Such a faulty restart is shown in Figs. 11 and 12. If a sheet of paper breaks and is not restarted, the pad is reduced and increases the voltage stress on the remaining plies.

7. Foreign Material in the Winding. Any foreign material found in the pack, and which is not made into the paper, comes under this category. A piece of such material may be sharp or smooth, it may be a good dielectric, or may be a conductor. A smooth piece of good dielectric is bad only insofar as it dimples a normally smooth pad and distorts the electric field due to a dielectric constant different from the oiled paper. If the material is sharp, it may puncture the pad, leading to a shorting failure. If the material is a conductor, it will shunt and distort the electric field; if it lies in a margin, it will have the effect of narrowing the margin at this point.

8. Solder Connections. Solder connections occur between the pack end foils and the collector web (D in Fig. 2) at both top and bottom of the packs and between the collector web and the header collector (in Fig. 2). A loose connection in one of these areas will cause sparking and resultant carbonization of the impregnant. A short to the case is the usual result. Because the carbon build-up from a single shot is small, many shots are usually required before this type of defect can cause a failure.

The pack can be damaged during soldering. The solder operation takes place only a fraction of an inch from the edge margins of the pack and if great care is not exercised the margin may be torn. A tear in the paper here will result in an edge margin failure.

9. Icicles. Icicles are stalactites of solder that drop down between or outside the packs either during the soldering of the headers or during sealing of the case. Since an icicle forms an electric field shunt, very high potentials may be obtained at the ends of the icicle. Because the pack has extra wraps of paper around it after the foils have ended, there is usually enough insulation to prevent breakdown if the icicle is short. If it is long, most of the voltage across the capacitor can occur at the ends of the icicle and a failure results. If the icicle is formed during sealing, it usually stays outside the major insulation and is not a problem. Sealing icicles are much more smooth and round than scrub-solder icicles since they are usually formed under oil.

10. Major Insulation. The major insulation wrapped around the packs, item E in Fig. 2, forms the insulation between the packs and the case. This insulation can be damaged by burning while the case is being welded shut; it can be torn before or during insertion into the case, or parts can be folded back during insertion to expose some of the packs to the case. On occasion, some of this insulation is left out during assembly. Damage to, or an insufficient amount of the major insulation can result in a short between the pack and the case.

11. Impregnation Cycle. Problems in the impregnation cycle are (1) overheating during the drying cycle, (2) over- and underdrying, (3) underimpregnation, and (4) underfilling with impregnant.

After the unit has been assembled, but before impregnation, it is heated under vacuum to remove excess moisture. If the temperature is too high, the paper will char, resulting in a brittle and weakened dielectric. If the drying cycle is too short, moisture will remain in the paper. This lowers the effectiveness of the impregnant, leads to more corona at the edges of the margin, and to excessive evolution of gas during operation. If the paper is overdried, it becomes brittle and may crack and break during flexing produced by temperature changes and electrical discharge. After drying, the impregnant is introduced into the case under vacuum. The unit is maintained at an elevated temperature during impregnation to lower the viscosity of the impregnant and speed its passage into the paper. If the vacuum is insufficient, complete impregnation may never take place. Similarly, if the vacuum is removed too soon, complete impregnation may not occur. During this portion of the cycle, the temperature must be held below a certain level, usually lower than the drying cycle temperature, to avoid damaging the impregnant. Underfilling occurs when insufficient impregnant is supplied to fill the case.

Overheating and overdrying are fairly easy to detect in a failed unit. In underdrying, one is given several clues by the life and manner of capacitor operation as well as the appearance of the pack margins, which usually show an abnormal amount of corona. Incomplete impregnation can be hard to detect since a dry patch often looks very much like the normal hydrogenation produced in the impregnant after the capacitor has been used for a few thousand shots. Incomplete impregnation can result in capacitor lives shortened to several thousand shots but not to immediate failures. If the case has not ruptured or swelled excessively, underfilling is easily checked by measuring the oil level prior to opening the case.

12. <u>Bushing</u>. Occasionally, the center insulating bushing (the area between A and B in Fig. 2) may be defective. This bushing must withstand the charge voltage as well as transient discharge voltages which can be 25% higher than the charge voltage. If the bushing fails, the capacitor must be discarded, even though the packs are still good.

The above listing of manufacturing deficiencies is probably not complete since it contains only those actually observed in one or more capacitors. Table I shows the number of times a particular type of deficiency was observed in a sample of 29 units from a single manufacturer. Cause of failure is given for only 25 capacitors in the table; the remaining four failures are considered unknown.

TABLE I

FAULTS FOUND IN A SAMPLE OF 29 CAPACITORS FROM A SINGLE MANUFACTURER

Fault Type	No. of Times Observed	No. of Faults Directly Attributable to Fault Type
1. Winding machine setup	23	3
2. Paper and foil wander	21	0
3. Fold-overs	11	5
4. Wrinkles and folds	14	4
5. Winding starts	8	3
6. Foil and paper breakage	6	2
7. Foreign material	0	0
8. Solder connections	16	5
9. Icicles	13	3
10. Major insulation	3	0
11. Impregnation	6	0
		25

IV. EFFECT OF WINDING VARIATIONS

The aim of this section is to examine certain design areas such as margin width, active area, and voltage stress to determine the effect of normal variations introduced during manufacture. We will require analytical expressions for the length of foil in a pack; the foil active width as a function of pad thickness and a number of series sections; and a relation between the total pack width, the number of series sections, and the margin widths. These will then be used to derive expressions for the maximum margin fields and pad voltages from a given pack configuration.

A. Calculation of the Single Foil Length in a Pack.

Assume the pack is wound on a mandril of radius r_m. Let r be the desired outer radius of the finished pack. The thickness, t, of each layer of winding is given by

$$t = 2t_f + 2t_p, \tag{1}$$

where t_f is the foil thickness and t_p is the pad thickness. The number of turns in each pack is given by

$$n = \frac{r - r_m}{t} . \tag{2}$$

The length of the ith turn is given approximately by

$$\ell_i = 2\pi\ r_i, \tag{3}$$

where r_i is the average radius of the ith turn (Fig. 13).

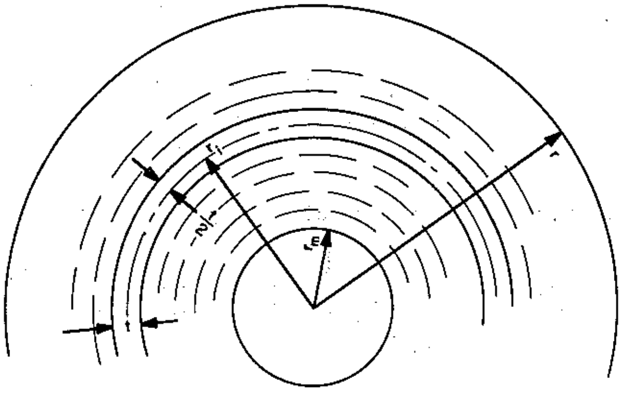

Fig. 13. Diagram of capacitor pack winding.

The total length of a single foil is then given by

$$L_o = \sum_1^n 2\pi\ r_i . \tag{4}$$

The radius, r_i, is given by

$$r_i = r_m + (i - \tfrac{1}{2})t \tag{5}$$

so we get the expression

$$\begin{aligned} L_o &= \sum_1^n 2\pi\ r_i = 2\pi \sum_1^n (r_m + (i - \tfrac{1}{2})t) \\ &= 2\pi n(r_m - t/2) + 2\pi t \sum_1^n i = 2\pi n(r_m - t/2) \\ &+ 2\pi t\left\{\frac{n}{2}\ (1 + n)\right\} = 2\pi n(r_m - t/2) + \pi t n(1 + n). \end{aligned} \tag{6}$$

Substituting $n = \frac{r - r_m}{t}$ (Eq. 2), we get

$$\begin{aligned} L_o &= 2\pi\frac{(r - r_m)}{t}\ (r_m - t/2) + \pi t \left(\frac{r - r_m}{t}\right) + 1 \\ &= \frac{\pi}{t}\ (r^2 - r^2_m), \end{aligned} \tag{7}$$

or using Eq. (1),

$$L_o = \frac{\pi(r^2 - r^2_m)}{2(t_f + t_p)} . \tag{8}$$

B. Foil Active Width As a Function of Pad Thickness, Foil Thickness, and the Number of Series Sections.

The capacity of a single pack is given by

$$C_p = \frac{C_t}{m} , \tag{9}$$

where C_t is the total capacity and m is the number of packs which can be put into the case.

Given C_p and the number of sections, the required active area per section and the pad thickness are related by

$$\frac{A}{t_p} = \frac{nC_p}{K} , \tag{10}$$

where A = active area per section,
t_p = the pad thickness,
n = the number of sections, and
K = a factor depending on the dielectric material used and is a constant for a given capacitor.

The active area is also given by

$$A = 2L_oW_o, \tag{11}$$

where

L_o = the foil length in Eq. (8) and

W_o = the foil active width.

The factor of 2 enters because both sides of the foil are active. Eliminating A between Eqs. (10) and (11) gives

$$\frac{2\ L_oW_o}{t_p} = \frac{nC_p}{K}, \tag{12}$$

and introducing L_o from Eq. (8) gives finally

$$W_o = \frac{nC_p\ t_p\ (t_p + t_t)}{\pi K(r^2 - r^2_m)}. \tag{13}$$

In general, the quantities r, r_m, C_p, K, and t_f are all fixed for a given pack so Eq. (13) shows that the required active width is proportional to the number of sections and to a quadratic term in the pad thickness, i.e.,

$$W_o \propto n(t^2_p + t_f\ t_p). \tag{14}$$

C. Relation Between the Total Pack Width, the Number of Series Sections, and the Margin Widths.

If D is the total pack width, we can write

$$D = nW_o + (n - 1)\ M_{oi} + 2\ M_{oe}, \tag{15}$$

where M_{oi} is the width of an interior margin and M_{oe} is the width of an end margin. The remaining symbols have the same meanings as before.

Equation (13) shows that the required active width per section increases with the number of sections. Equation (15) shows that the product nW_o is limited by D and the required margin widths. We now determine the effect of variations in the active width. This is best approached by computing the actual voltage and static field distribution in a given pack.

D. Voltage and Field Distribution in an Arbitrary Pack.

(The notation is that of Fig. 3) In any capacitor pack with n sections and n + 1 margins, the voltage across the pad at each section is given by

$$V_{s(i)} = \frac{Q_p}{C_i}, \tag{16}$$

where C_i is the capacity of the ith section, Q_p is the charge on the pack, and $V_{s(i)}$ is the voltage across the section.

The total capacity of the pack is given by

$$C_p = \frac{1}{\sum\limits_{1}^{n} 1/C_i}. \tag{17}$$

The charge on this capacity is given by

$$Q_p = V_o\ C_p, \tag{18}$$

where V_o is the total voltage across the pack. The voltage across the ith margin is

$$V_{m(i)} = V_{s(i)} + V_{s(i\ -\ 1)}, \tag{19}$$

if we adopt the convention that $V_{s(o)} = V_{s(n\ +\ 1)} = 0$. The capacity per section is

$$C_i = K_o\ W_i\ L_i. \tag{20}$$

Here, L_i is the length of active foil in the ith section and K_o is the constant for a given capacitor. Substituting Eq. (20) into Eq. (17), we get

$$C_p = \frac{K_o}{\sum\limits_{i=1}^{n} \frac{1}{W_iL_i}} \tag{21}$$

and inserting this into Eq. (18), we have

$$Q_p = V_o \frac{K_o}{\sum\limits_{i=1}^{n} \frac{1}{W_iL_i}} \tag{22}$$

Using Eq. (20) and Eq. (22) in Eq. (16), we finally have

$$V_{s(j)} = V_o \frac{1}{\sum\limits_{i=1}^{n} \frac{1}{W_iL_i}} \times \frac{1}{W_j\ L_j}. \tag{23}$$

This gives us, for the voltage across the jth margin,

$$V_{m(j)} = \frac{V_o}{\sum\limits_{i=1}^{n} \frac{1}{W_iL_i}} \left\{ \frac{1}{W_j\ L_j} + \frac{1}{W_{j-1}\ L_{j-1}} \right\}. \tag{24}$$

The average electric field across the jth margin is

$$E_j = \frac{V_{m(j)}}{M_j} = \frac{V_o}{M_j \sum_{i=1}^{n} \frac{1}{W_i\ L_i}} \left(\frac{1}{W_j\ L_j} + \frac{1}{W_{j-1}\ L_{j-1}} \right), \quad (25)$$

where M_j is the width of the jth margin. Equations (23) and (25) can be used, a postiori, to determine the actual pad voltage and margin field in a failed pack. Usually, in any pack, all the foil lengths are equal, and if this is true, we get the simpler equations

$$V_{s(j)} = \frac{V_o}{\sum_{i=1}^{n} \frac{1}{W_i}} \times \frac{1}{W_j} \quad (23)'$$

and

$$E_j = \frac{V_o}{M_j \sum_{i=1}^{n} \frac{1}{W_i}} \left(\frac{1}{W_j} + \frac{1}{W_{j-1}} \right). \quad (25)'$$

Equations (23) and (25) can also be used to evaluate a proposed design. To do this, we note that Eq. (23) can be written

$$\frac{V_{s(j)}}{V_{so}} = \frac{W_o L_o}{W_j\ L_j} \times \frac{n}{\sum_{i=1}^{n} \frac{W_o L_o}{W_i L_i}} \quad (26)$$

where

$$L_i = L_o + \Delta L_i,$$

$$W_i = W_o + \Delta W_i,$$

and V_{so} is the section design voltage.

For a design, we can assume that ΔL_i is negligible so the maximum value of the expression

$$\frac{V_{s(j)}}{V_{so}} = \frac{W_o}{W_j} \times \frac{n}{\sum_{i=1}^{n} \frac{W_o}{W_i}} \quad (27)$$

subject to the condition

$$\frac{W_{min}}{W_o} \le \frac{W_i}{W_o} \le \frac{W_{max}}{W_o}, \quad (28)$$

is the maximum of the ratio of the maximum-section voltage to the section-design voltage which would be expected to appear in a pack due to active-width variations. Condition (28) can be determined from actual measurements; that is, this condition is determined by a winding machine-operator team.

Writing Eq. (27) in the form

$$\frac{V_{s(j)}}{V_{so}} = \frac{n}{1 + \sum_{\substack{i=1 \\ i \ne J}}^{n} \frac{W_j}{W_i}} = \frac{n}{1 + \sum_{\substack{i=1 \\ i \ne J}}^{n} \frac{W_o + \Delta W_j}{W_o + \Delta W_i}}, \quad (29)$$

we see that the maximum value is achieved when

$$\Delta W_i = \Delta W \qquad i \ne j \quad (30)$$

$$\Delta W_j = - \Delta W,$$

where ΔW is the maximum positive active width variation allowed by condition (28). Condition (30) is possible since Eq. (15) only limits the pack length; the end foils must always extend past D in order to make electrical connections, hence the foils are allowed a much greater range of movement than seems apparent at first.

Thus,

$$\left(\frac{V_{s(j)}}{V_{so}} \right)_{max} = \frac{n}{1 + \frac{(W_o - \Delta W)(n-1)}{W_o + \Delta W}} = \frac{1 + \frac{\Delta W}{W_o}}{1 - \left(1 - \frac{2}{n}\right) \frac{\Delta W}{W_o}}. \quad (31)$$

$\left(\frac{V_{s(j)}}{V_{so}} \right)_{max}$ is plotted vs $\frac{\Delta W}{W_o}$ for n values ranging from 3 to 10 in Fig. 14. The expression shows that

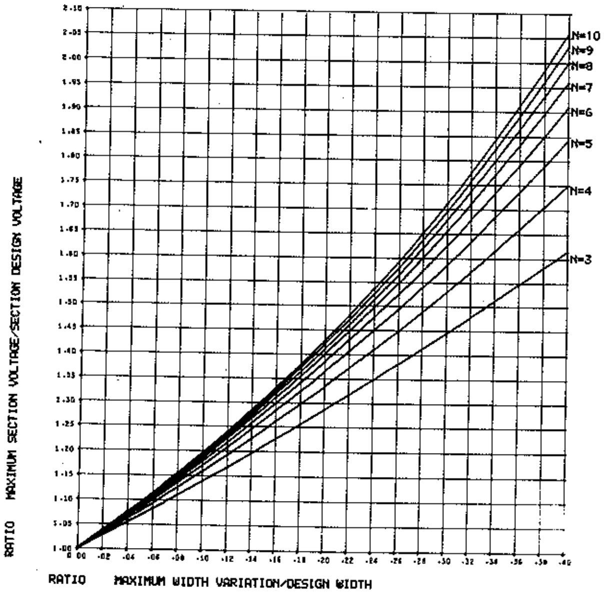

Fig. 14. Plots of Eq. (31) for several numbers of sections.

if $\frac{\Delta W}{W_o}$ is large, quite large increases in voltage can occur. Since ΔW is a fixed value, this shows that W_o should be kept as large as possible subject to the restriction imposed by Eq. (15).

It was felt that the maximum given in Fig. 14 might be much higher than the maximum actually occurring in a given pack; however, a check made of a group of ten-section packs showed that the actual maximum-section-voltage to section-design-voltage ratio was in every case greater than 75% of the value obtained from Fig. 14 when the maximum width variation, found in the pack itself, was used to determine the ratio. In one case, the actual value was 98% of the absolute maximum.

Equation (31) does not account for variations in the thickness of the paper dielectric. Such variations will increase the local electric field. Since the maximum variation is ± 20%, we can expect a maximum electric field increase of 25%.

From Eq. (25), with the condition that all foil lengths are equal, we find that the maximum margin field occurs at the maximum of

$$E_j = \frac{n\,V_{os}}{M_j \sum_{k=1}^{n} \frac{1}{W_k}} \times \left(\frac{1}{W_i} + \frac{1}{W_{i-1}}\right) . \tag{32}$$

To make Eq. (32) dimensionless, we need an expression for the design margin field, E_o:

$$E_o = \frac{2V_{os}}{M_o} . \tag{33}$$

Thus, we wish to maximize the expression

$$\frac{E_i}{E_o} = \frac{M_o\,n}{2(M_o + \Delta M_i)} \times \frac{1}{\sum_{k=1}^{n} \frac{1}{W_o + \Delta W_k}} \times \left(\frac{1}{W_o + \Delta W_i} + \frac{1}{W_o + \Delta W_{i-1}}\right), \tag{34}$$

where we have made the substitution

$M_i = M_o + \Delta M_i$

$W_i = W_o + \Delta W_i$.

Only one margin variation appears in Eq. (34) so the maximum of Eq. (34) will occur when $\Delta M_i = - \Delta M$, (ΔM being the maximum margin variation that is produced by the winding technique) and when the expression

$$F = \frac{1}{\sum_{k=1}^{n} \frac{1}{W_o + \Delta W_k}} \times \left\{\frac{1}{W_o + \Delta W_i} + \frac{1}{W_o + W_{i-1}}\right\} \tag{35}$$

has a maximum subject to the constraint of Eq. (28). If we write Eq. (35) in the form

$$F = \left\{\frac{1}{1 + (W_o + \Delta W_i) \sum_{\substack{k=1 \\ k\neq 1,i-1}}^{n} \frac{1}{W_o + \Delta W_k} + \frac{W_o + \Delta W_i}{W_o + \Delta W_{i-1}}} + \frac{1}{1 + (W_o + \Delta W_{i-1}) \sum_{\substack{k=1 \\ k\neq 1,i-1}}^{n} \frac{1}{W_o + \Delta W_k} + \frac{W_o + \Delta W_{i-1}}{W_o + \Delta W_i}}\right\} \tag{36}$$

it is obvious that F will be greatest if $\Delta W_k = \Delta W$ for $k \neq 1, i - 1$. Using this value, we obtain the expression

$$F = \left\{\frac{1}{1 + \frac{n-2}{1 + \frac{\Delta W}{W_o}}\ 1 + \frac{\Delta W_i}{W_o} + \frac{1 + \frac{\Delta W_i}{W_o}}{1 + \frac{\Delta W_{i-1}}{W_o}}} + \frac{1}{1 + \frac{n-2}{1 + \frac{\Delta W}{W_o}}\ \ 1 + \frac{\Delta W_{i-1}}{W_o}\ \ \frac{1 + \frac{\Delta W_{i-1}}{W_o}}{1 + \frac{\Delta W_i}{W_o}}}\right\} . \tag{37}$$

The relative maximum value of F, subject to Eq. (28), occurs when

$$\frac{\Delta W_i}{W_o} = \frac{\Delta W_{i-1}}{W_o} = - \frac{\Delta W}{W_o} ,$$

and then

$$F_{Max} = \frac{2}{2 + (n - 2) \frac{1 - \frac{\Delta W}{W_o}}{1 + \frac{\Delta W}{W_o}}} . \tag{38}$$

Values of this maximum as a function of $\frac{\Delta W}{W_o}$ for $n = 3$ to 10 are given in Table II.

TABLE II

TABLE OF RELATIVE MAXIMA OF F (EQ. 38)

ΔW/W	3	4	5	6	7	8	9	10
0.01	0.671	0.505	0.405	0.338	0.290	0.254	0.226	0.203
0.02	0.675	0.510	0.410	0.342	0.294	0.258	0.229	0.206
0.03	0.680	0.515	0.414	0.347	0.298	0.261	0.233	0.209
0.04	0.684	0.520	0.419	0.351	0.302	0.265	0.236	0.213
0.05	0.689	0.525	0.424	0.356	0.307	0.269	0.240	0.216
0.06	0.693	0.530	0.429	0.361	0.311	0.273	0.244	0.220
0.07	0.697	0.535	0.434	0.365	0.315	0.277	0.247	0.223
0.08	0.701	0.540	0.439	0.370	0.320	0.281	0.251	0.227
0.09	0.706	0.545	0.444	0.375	0.324	0.285	0.255	0.230
0.10	0.710	0.550	0.449	0.379	0.328	0.289	0.259	0.234
0.11	0.714	0.555	0.454	0.384	0.333	0.294	0.263	0.238
0.12	0.718	0.560	0.459	0.389	0.337	0.298	0.267	0.241
0.13	0.722	0.565	0.464	0.394	0.342	0.302	0.271	0.245
0.14	0.726	0.570	0.469	0.399	0.347	0.306	0.275	0.249
0.15	0.730	0.575	0.474	0.404	0.351	0.311	0.279	0.253
0.16	0.734	0.580	0.479	0.408	0.356	0.315	0.283	0.257
0.17	0.738	0.585	0.484	0.413	0.361	0.320	0.287	0.261
0.18	0.742	0.590	0.490	0.418	0.365	0.324	0.291	0.265
0.19	0.746	0.595	0.495	0.423	0.371	0.329	0.296	0.269
0.20	0.750	0.600	0.500	0.429	0.375	0.333	0.300	0.272

Finally, the expression for the maximum expected margin field is given as

$$\left(\frac{E_i}{E_o}\right)_{max} = \frac{n}{2\left(1 - \frac{\Delta M}{M_o}\right)} F_{max}\left(n, \frac{\Delta W}{W}\right). \qquad (38)$$

Let us apply Eqs. (31) and (38) to an actual design in which

$n = 9$
$W_o = 1.75$
$M_o = 0.75$
$\Delta W = 0.14$
$\Delta M = 0.09$

therefore,

$$\frac{\Delta W}{W_o} = 0.08.$$

From Eq. (31) we find

$$\left(\frac{V_{s(j)}}{V_{s(o)}}\right)_{max} = \frac{1 + \frac{0.14}{1.75}}{1 - (1 - 2/9)\,\frac{0.14}{1.74}} = 1.15,$$

and from Eq. (38) and Table II we get

$$\left(\frac{E_i}{E_o}\right)_{max} = \frac{9}{2\left(1 - \frac{0.09}{0.75}\right)} (0.251) = 1.28.$$

In this instance, we expect, in the worst cases, about a 15% maximum pad overvoltage and a 28% margin field increase. A direct calculation of sample packs, taken from units of this design, gave $\left(\frac{V_{s(j)}}{V_{s(o)}}\right)_{max} = 1.08$. This is 94% of the worst case value given by Eq. (31) and $\left(\frac{E_i}{E_o}\right)_{max} = 1.14$, 89% of the worst case value given by Eq. (38). In this case, the choice between E_o is the more critical; that is, it should be chosen conservatively to prevent margin failures. In the particular case of the units used in the example, E_o was not chosen conservatively. These units had a very short life and the failures all appeared to start across margins.

V. CONCLUSIONS

It is possible in many cases of capacitor failure to pinpoint the cause by careful examination of the failed unit. Such examinations can also be used to define the problem areas of a particular manufacturer. The equations derived in Sec. IV allow a proposed design to be checked prior to manufacture to see if the maximum allowable pad voltage will be exceeded or if the margin fields will be too high.

VI. ACKNOWLEDGMENTS

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